JOURNAL OF GUIDANCE, CONTROL, AND DYNAMICS
Vol. 18, No. 4, July–August 1995

# Nonlinear Decoupling Control of Aircraft Motion

Zhou Zhiqiang*
*Northwestern Polytechnical University, Shaanxi 710072, People's Republic of China*

**In this paper, decoupling theory for nonlinear multivariable systems based on differential geometry control theory is introduced. The influence of position exchange in components of input and decoupling variable on the decoupling law of the nonlinear system is discussed. The calculation of equilibrium points for nonlinear decoupling systems is also studied. For the engineering application, linear approximation of the aircraft nonlinear decoupling law is studied. The error made by the approximation is acceptable. Three types of decoupling controls for aircraft nonlinear motion are studied. Three basic modes of nonlinear direct force control and two types of extraordinary maneuvers are realized through the nonlinear decoupling controls. The calculation results show that the nonlinear decoupling control law can realize the modes of nonlinear direct force control and the extraordinary maneuvers thoroughly.**

## Nomenclature

| | |
|---|---|
| $b$ | = wing span, m |
| $\bar{c}$ | = wing mean chord, m |
| $g$ | = gravitational acceleration, m/s$^2$ |
| $I_x, I_y, I_z, I_{xz}$ | = moments and product of inertia about body axes, kg-m$^2$ |
| $L, M, N$ | = rolling, pitching, and yawing moments about body axes, kg-m$^2$/s$^2$ |
| $p, q, r$ | = angular velocity components along roll, pitch, and yaw axes of aircraft, deg/s |
| $\bar{q}$ | = $\frac{1}{2}\rho V^2$, kg/m-s$^2$ |
| $S$ | = wing plan area, m$^2$ |
| $T$ | = thrust, kg-m/s$^2$ |
| $V$ | = velocity of aircraft center of mass, m/s |
| $X, Y, Z$ | = aerodynamic forces in body axes, kg-m/s$^2$ |
| $x, y, z$ | = body axes of aircraft |
| $\alpha$ | = angle of attack, deg |
| $\beta$ | = angle of sideslip, deg |
| $\gamma$ | = angle of climb, deg |
| $\delta_a, \delta_r, \delta_e$ | = aileron, rudder, and elevator deflection angles, deg |
| $\delta_c, \delta_f$ | = deflection angles of direct side force and direct lift control surfaces, deg |
| $\delta_T$ | = throttle |
| $\theta$ | = pitch angle, deg |
| $\sigma$ | = angle between thrust axis and body $x$ axis, deg |
| $\varphi$ | = angle of bank, deg |
| $\psi$ | = azimuth angle, deg |

*Subscripts*

| | |
|---|---|
| $p, q, r$; $\delta_a, \delta_r, \delta_e$; $\delta_c, \delta_f, \delta_T$ | = denote partial derivatives with respect to respective quantity |
| $e$ | = equilibrium point |

## Introduction

ONE of the main functions in active control technology (ACT) is direct force control, which decouples parts or all of the motion parameters of the aircraft with the direct force control surfaces in order to improve flight performance and flying qualities. Therefore, direct force control can be characterized as decoupling control of multivariable systems.

With research on fourth-generation combat aircraft and the appearance of the all-aspect missile, the fighter with the ability of decoupling between heading angle and flight path will be of benefit in air combat. The decoupling control of the aircraft motion is one of the means to increase aircraft agility. Thus, decoupling control of aircraft flight provides a new theoretical approach to investigate direct force control and aircraft agility.

In recent years, several researchers have applied nonlinear decoupling theory and dynamic inversion approach to design control systems for aircraft.[1–3] In a slightly different approach, the flight control system of a helicopter is designed using exact linearization theory by state feedback and nonlinear transformation.[4] A singular perturbation approach has been used in Ref. 5 to avoid difficulty due to near singularity in the input–output map. Zero dynamics for a flight control system are obtained in Ref. 6 in order to examine the stability of the decoupled system. An application of nonlinear inversion theory to the design of flight control systems is presented in Ref. 7. The synthesis procedures are similar to those in Ref. 1. One of the sets of output variables chosen in Ref. 7 is $(\varphi, \alpha, \beta)$ control, which is also used in this paper. However, the control laws derived and the model of the aircraft here are different. The control system is divided into inner and outer control loops in Ref. 7, but the control laws here are derived directly based on decoupling control theory.

Application of decoupling theory for nonlinear multivariable systems based on differential geometry control theory[8] is presented here. Three sets of output variables are chosen, which differ from those of Refs. 1–6. Three types of decoupling controls corresponding to three sets are used to investigate direct force control and two types of aircraft agility maneuvers. The influence of position exchange in components of input and decoupling variable on the nonlinear decoupling law of nonlinear systems is given. The calculation of equilibrium points for nonlinear decoupling systems and linear approximation of the aircraft nonlinear decoupling law are also studied here.

## Synthesis of Nonlinear Decoupling Control Law

The nonlinear state equation of the aircraft motion can be described by

$$\dot{x} = A(x) + B(x)u, \qquad y = C(x) \tag{1}$$

where $A(x)$, $B(x)$, and $C(x)$ are an $n \times 1$ vector, $n \times m$ matrix, and $m \times 1$ vector, respectively, as functions of state variable $x$.

Consider a nonlinear control law, which is composed of a nonlinear feedback of state variables and a nonlinear input transformation as follows:

$$u = F(x) + G(x)v \tag{2}$$

Received May 20, 1994; presented as Paper 94-3517 at the AIAA Atmospheric Flight Mechanics Conference, Scottsdale, AZ, Aug. 1–3, 1994; revision received Oct. 25, 1994; accepted for publication Oct. 25, 1994.


*Lecturer, Aircraft Engineering Department.

where $F(x)$ and $G(x)$ are $m \times 1$ and $m \times m$ matrices, respectively, and $G(x)$ is assumed to be nonsingular.

Applying the control law (2) to the system Eq. (1) gives the closed-loop system

$$\dot{x} = \hat{A}(x) + \hat{B}(x)v, \qquad y = C(x) \tag{3}$$

where

$$\hat{A}(x) = A(x) + B(x)F(x), \qquad \hat{B}(x) = B(x)G(x)$$

The closed-loop system (3) is said to be decoupled by the nonlinear control law (2) if the $i$th input $v_i$ affects only the $i$th output $y_i$ for all $i = 1, 2, \ldots, m$, where $v_i$ and $y_i$ are the $i$th components of $v$ and $y$, respectively.

Based on differential geometric control theory,[8] the nonlinear decoupling control problem of the nonlinear system (1) by means of a static feedback [Eq. (2)] is solvable if and only if the $m \times m$ matrix

$$B^*(x) = \begin{bmatrix} L_{b_1} L_A^{r_1-1} c_1(x) & \cdots & L_{b_m} L_A^{r_1-1} c_1(x) \\ \vdots & & \vdots \\ L_{b_1} L_A^{r_m-1} c_m(x) & \cdots & L_{b_m} L_A^{r_m-1} c_m(x) \end{bmatrix} \tag{4}$$

which is defined for all $x$ in a neighborhood of an initial point $x_0$, is nonsingular for all $x$ in the neighborhood of the point $x_0$.

A multivariable nonlinear system of the form of Eq. (1) has a vector relative degree $\{r_1, r_2, \ldots, r_m\}$ at a point $x_0$ if 1) $L_{b_j} L_A^k c_i(x) = 0$ for all $1 \le j \le m$, for all $1 \le i \le m$, for all $k < r_i - 1$, and for all $x$ in a neighborhood of $x_0$, and 2) the $m \times m$ matrix $B^*(x)$ is nonsingular at $x = x_0$.

The definition of the differential operator $L_A c_i(x)$ is as follows. If $\lambda$ is a real-valued function and $f$ is a vector field, then

$$L_f \lambda(x) = \sum_{i=1}^{n} \frac{\partial \lambda}{\partial x_i} f_i(x)$$

The function $L_f^k \lambda(x)$ satisfies the recursion

$$L_f^k \lambda(x) = \frac{\partial \left(L_f^{k-1} \lambda\right)}{\partial x} f(x)$$

with $L_f^0 \lambda(x) = \lambda(x)$.

If $B^*(x)$ is nonsingular at $x_0$, $F(x)$ and $G(x)$ in the nonlinear decoupling law (2) are given by

$$\begin{aligned} F(x) &= -B^{*^{-1}}(x)[A^*(x) + D^*(x)] \\ G(x) &= -B^{*^{-1}}(x)\wedge \end{aligned} \tag{5}$$

where $\wedge = \mathrm{diag}\{\lambda_1, \lambda_2, \ldots, \lambda_m\}$ ($\lambda_i \neq 0$, $i = 1, 2, \ldots, m$) is a constant matrix, and

$$A^*(x) = \left(L_A^{r_1} c_1(x) \quad \cdots \quad L_A^{r_i} c_i(x) \quad \cdots \quad L_A^{r_m} c_m(x)\right)^T$$

$$D^*(x) =$$

$$\begin{bmatrix} \alpha_{1r_1-1} L_A^{r_1-1} c_1(x) + \cdots + \alpha_{11} L_A c_1(x) + \alpha_{10} c_1(x) \\ \vdots \\ \alpha_{ir_i-1} L_A^{r_i-1} c_i(x) + \cdots + \alpha_{i1} L_A c_i(x) + \alpha_{i0} c_i(x) \\ \vdots \\ \alpha_{mr_m-1} L_A^{r_m-1} c_m(x) + \cdots + \alpha_{m1} L_A c_m(x) + \alpha_{m0} c_m(x) \end{bmatrix}$$

where $\alpha_{ir_i-1}, \alpha_{ir_i-2}, \ldots, \alpha_{i1}, \alpha_{i0}$ ($i = 1, 2, 3, \ldots, m$) are constants, which can be chosen to specify the dynamic behavior of the decoupled closed-loop system. The output vector $y$ satisfies the following differential equation:

$$y_i^{(r_i)} + \alpha_{ir_i-1} y_i^{(r_i-1)} + \alpha_{ir_i-2} y_i^{(r_i-2)} + \cdots + \alpha_{i1} y_i^{(1)} + \alpha_{i0} y_i = \lambda_i v_i$$

$$i = 1, 2, \ldots, m \tag{6}$$

The influence of position exchange in components of the input and the decoupling variable on the nonlinear decoupling law and the closed-loop system is given by the following two theorems.

*Theorem 1.* When the component $y_i$ ($i = 1, 2, \ldots, m$) of $y(x)$ exchanges its position, the feedback matrix $F(x)$ in Eq. (5) remains unchanged, and the column of matrix $G(x)$ in Eq. (5) exchanges its position in the same way as $y_i$; $\hat{A}(x)$ in the closed-loop system (3) remains also unchanged and the column of matrix $\hat{B}(x)$ exchanges its position similar to $G(x)$.

*Theorem 2.* When the component $u_i$ ($i = 1, 2, \ldots, m$) of $u$ exchanges its position, the row of the matrices $F(x)$ and $G(x)$ in Eq. (5) exchanges its position, respectively, in the same way as $u_i$; matrices $\hat{A}(x)$ and $\hat{B}(x)$ remain unchanged.

If the control law given by Eq. (5) for output $y_A = (c_1(x), c_2(x), \ldots, c_n(x))^T$ is known, for example, it is very easy to obtain the control law for output $y_B = (c_2(x), c_1(x), \ldots, c_n(x))^T$ from the control law for output $y_A$ according to Theorem 1.

## Equilibrium Points of Nonlinear Decoupling System

Let $x_e$ be one of the equilibrium points of the nonlinear system (1). Here, $u_e$ is the input of the system of Eq. (1) corresponding to $x_e$. For the nonlinear decoupled closed-loop system (3), $v_e$ of the new input $v$ corresponding to the equilibrium $x_e$ is given by

$$v_e = \wedge^{-1}[B^*(x_e)u_e + A^*(x_e) + D^*(x_e)] \tag{7}$$

or

$$v_e = \wedge^{-1} B^*(x_e)[u_e - F(x_e)] \tag{8}$$

The equilibrium value $y_e$ of the output $y$ is given by

$$y_e = C(x_e)$$

Then, $v_e$ can be also obtained by

$$v_{ie} = \alpha_{i0} y_{ie} / \lambda_i, \qquad i = 1, 2, \ldots, m \tag{9}$$

The equilibrium points of the nonlinear decoupled system were studied by the author for the first time. The study not only has theoretical significance but is also meaningful for engineering problems. Because the engineering problems are normally complicated, the nonlinear decoupling control law of Eq. (5) can be obtained mostly by numerical calculation with computers. Equation (7) can be used to verify the correctness of the numerical results.

## Approximate Solution of Nonlinear Decoupling Control Law

The decoupling control law of Eq. (5) for the aircraft in large maneuvers is nonlinear and can be generally obtained only through numerical calculations. If the on-line decoupling control law is required, a large amount of numerical calculations will be the main obstacle for realization. One way to reduce the amount of calculations is the search for an approximate solution of the nonlinear decoupling control law.

Let $\tilde{F}$ and $\tilde{G}$ be the approximate solutions of $F(x)$ and $G(x)$, respectively. Then $\tilde{F}$ and $\tilde{G}$ are given by

$$\tilde{F} = \left.\frac{\partial F(x)}{\partial x}\right|_{x=x_e}, \qquad \tilde{G} = G(x)|_{x=x_e} \tag{10}$$

where $x_e$ is one of the equilibrium points of the decoupled closed-loop system given by Eq. (3).

## Nonlinear Decoupling Control of Aircraft Motion

According to the above-mentioned theory, three modes of aircraft decoupling control are investigated. The nonlinear decoupling laws of an aircraft are derived for the modes described below:

Mode *A*

$$y = \begin{bmatrix} \gamma \\ \theta \end{bmatrix}, \qquad u = \begin{bmatrix} \delta_f \\ \delta_e \end{bmatrix} \tag{11}$$

Mode $B$

$$y=\begin{bmatrix}\alpha\\ \theta\end{bmatrix},\qquad u=\begin{bmatrix}\delta_f\\ \delta_e\end{bmatrix}\tag{12}$$

Mode $C$

$$y=\begin{bmatrix}\alpha\\ \beta\\ \varphi\end{bmatrix},\qquad u=\begin{bmatrix}\delta_e\\ \delta_r\\ \delta_a\end{bmatrix}\tag{13}$$

The basic equations of motion for a rigid-body aircraft are given by

$$\dot V=\left(\frac{X+T\cos\sigma}{m}-g\sin\theta\right)\cos\alpha\cos\beta$$
$$+\left(\frac{Y}{m}+g\cos\theta\sin\varphi\right)\sin\beta$$
$$+\left(\frac{Z-T\sin\sigma}{m}+g\cos\theta\cos\varphi\right)\sin\alpha\cos\beta$$

$$\dot\alpha=q-\frac{[(X+T\cos\sigma)/mV-(g/V)\sin\theta+r\sin\beta]\sin\alpha}{\cos\beta}$$
$$+\frac{[(Z-T\sin\sigma)/mV+(g/V)\cos\theta\cos\varphi-p\sin\beta]\cos\alpha}{\cos\beta}$$

$$\dot\beta=-\left[\left(\frac{X+T\cos\sigma}{mV}-\frac{g}{V}\sin\theta\right)\sin\beta+r\right]\cos\alpha$$
$$+\left(\frac{Y}{mV}+\frac{g}{V}\cos\theta\sin\varphi\right)\cos\beta$$
$$-\left[\left(\frac{Z-T\sin\sigma}{mV}+\frac{g}{V}\cos\theta\cos\varphi\right)\sin\beta-p\right]\sin\alpha$$

$$\dot p=\left[\left(\frac{I_y-I_z}{I_x}-\frac{I_{xz}^2}{I_xI_z}\right)qr+\left(1-\frac{I_y-I_x}{I_z}\right)\frac{I_{xz}}{I_x}pq\right.$$
$$\left.+\frac{1}{I_x}\left(L+\frac{I_{xz}}{I_z}N\right)\right]\frac{1}{D}$$

$$\dot q=\frac{1}{I_y}(M+M_T)+\frac{I_z-I_x}{I_y}pr+\frac{I_{xz}}{I_y}(r^2-p^2)$$

$$\dot r=\left[\left(\frac{I_{xz}^2}{I_xI_z}-\frac{I_y-I_x}{I_z}\right)pq+\left(\frac{I_y-I_z}{I_x}-1\right)\frac{I_{xz}}{I_z}qr\right.$$
$$\left.+\frac{1}{I_z}\left(\frac{I_{xz}}{I_x}L+N\right)\right]\frac{1}{D}$$

where

$$D=1-\left(I_{xz}^2/I_xI_z\right)$$

It is necessary to add three kinematic equations:

$$\dot\theta=q\cos\varphi-r\sin\varphi$$
$$\dot\varphi=p+(q\sin\varphi+r\cos\varphi)\tan\theta$$
$$\dot\psi=(q\sin\varphi+r\cos\varphi)/\cos\theta$$

The aerodynamic forces and moments as well as the force and moment created by engine thrust are defined in the following expansions:

$$X=\bar qS\left[C_x(\alpha,\ \beta)+C_{x\delta_f}(\alpha)\delta_f+C_{x\delta_e}(\alpha)\delta_e\right]$$
$$Y=\bar qS\left[C_y(\alpha,\ \beta)+C_{y\delta_a}(\alpha)\delta_a+C_{y\delta_r}(\alpha)\delta_r+C_{y\delta_c}(\alpha)\delta_c\right]$$
$$Z=\bar qS\left[C_z(\alpha,\ \beta)+C_{z\delta_f}(\alpha)\delta_f+C_{z\delta_e}(\alpha)\delta_e\right]$$
$$T=T_0+T_{\delta_T}\delta_T$$

$$L=\bar qSb\left\{C_l(\alpha,\ \beta)+\frac{b}{2V}\left[C_{l_r}(\alpha)r+C_{l_p}(\alpha)p\right]+C_{l\delta_a}(\alpha)\delta_a\right.$$
$$\left.+C_{l\delta_r}(\alpha)\delta_r+C_{l\delta_c}(\alpha)\delta_c\right\}$$

$$M=\bar qS\bar c\left[C_m(\alpha,\ \beta)+\frac{\bar c}{2V}C_{mq}(\alpha)q\right.$$
$$\left.+C_{m\delta_e}(\alpha)\delta_e+C_{m\delta_f}(\alpha)\delta_f\right]$$

$$N=\bar qSb\left\{C_n(\alpha,\ \beta)+\frac{b}{2V}\left[C_{n_r}(\alpha)r+C_{np}(\alpha)p\right]+C_{n\delta_a}(\alpha)\delta_a\right.$$
$$\left.+C_{n\delta_r}(\alpha)\delta_r+C_{n\delta_c}(\alpha)\delta_c\right\}$$

$$M_T=M_0+M_{\delta_T}\delta_T$$

As an example, the nonlinear decoupling control law given by Eq. (5) for mode $A$ is derived as follows.

For mode $A$, $A(x)$, $B(x)$, and $C(x)$ in the nonlinear state equation given by Eq. (1) are defined as

$$A(x)=(A(x)_1,A(x)_2,A(x)_3,A(x)_4)^T$$
$$=\begin{bmatrix}-a_{11}C_D+a_{12}\cos(\alpha+\sigma)+g\sin(\alpha-\theta)\\ q-a_{21}C_L-a_{22}\sin(\alpha+\sigma)+\dfrac{g}{V}\cos(\alpha-\theta)\\ a_{31}C_m+a_{32}C_{mq}q+a_{33}\\ q\end{bmatrix}\tag{14}$$

$$B(x)=(b_1,b_2)=\begin{bmatrix}\tilde C_{x\delta_f}&\tilde C_{x\delta_e}\\ \tilde C_{z\delta_f}&\tilde C_{z\delta_e}\\ \tilde C_{m\delta_f}&\tilde C_{m\delta_e}\\ 0&0\end{bmatrix}\tag{15}$$

$$y=C(x)=(\gamma,\theta)^T=(\theta-\alpha,\ \theta)^T\tag{16}$$

where

$$x=\{V,\alpha,q,\theta\}^T,\qquad u=\{\delta_f,\delta_e\}^T$$
$$a_{11}=\frac{\bar qS}{m},\qquad a_{12}=\frac{T_0}{m}$$
$$a_{21}=\frac{\bar qS}{mV},\qquad a_{22}=\frac{T_0}{mV}$$
$$a_{31}=\frac{\bar qS\bar c}{I_y},\qquad a_{32}=\frac{\bar qS\bar c}{I_y}\frac{\bar c}{2V},\qquad a_{33}=\frac{M_{T_0}}{I_y}$$
$$\tilde C_{x\delta_f}=-\frac{\bar qS}{m}C_{D\delta_f},\qquad \tilde C_{x\delta_e}=-\frac{\bar qS}{m}C_{D\delta_e}$$
$$\tilde C_{z\delta_f}=-\frac{\bar qS}{mV}C_{L\delta_f}$$
$$\tilde C_{z\delta_e}=-\frac{\bar qS}{mV}C_{L\delta_e},\qquad \tilde C_{m\delta_f}=\frac{\bar qS\bar c}{I_y}C_{m\delta_f}$$
$$\tilde C_{m\delta_e}=\frac{\bar qS\bar c}{I_y}C_{m\delta_e}$$

The nonlinear decoupling law for mode $A$ is obtained as follows:

$$B^*(x)^{-1} = \left[\begin{array}{cc} \tilde{C}_{m\delta_e} & \tilde{C}_{z\delta_e} \\ -\tilde{C}_{m\delta_f} & -\tilde{C}_{z\delta_f} \end{array}\right] \Big/ \Delta$$

$$\Delta = \det B^*(x) = \tilde{C}_{m\delta_f}\tilde{C}_{z\delta_e} - \tilde{C}_{m\delta_e}\tilde{C}_{z\delta_f}$$

$$A^*(x) = \left[\begin{array}{c} a_{21}C_L + a_{22}\sin(\alpha+\sigma) + \dfrac{g}{V}\cos(\alpha-\theta) \\ a_{31}C_m + a_{32}C_{mq}q + a_{33} \end{array}\right]$$

$$D^*(x) = \left[\begin{array}{c} \alpha_{10}(\theta-\alpha) \\ \alpha_{20}\theta + \alpha_{21}q \end{array}\right]$$

$$\wedge = \left[\begin{array}{cc} \lambda_1 & 0 \\ 0 & \lambda_2 \end{array}\right]$$

The output vector $y$ of the decoupled closed-loop system satisfies the following:

$$\dot{\gamma} + \alpha_{10}\gamma = \lambda_1 v_1, \qquad \ddot{\theta} + \alpha_{21}\dot{\theta} + \alpha_{20}\theta = \lambda_2 v_2$$

The equilibrium input $v_e$ required for a desired $\gamma_e$ and $\theta_e$ is given by

$$v_{1e} = \frac{\gamma_e\alpha_{10}}{\lambda_1} \qquad v_{2e} = \frac{\theta_e\alpha_{20}}{\lambda_2}$$

Modes $A$ and $B$ can be used to realize the three basic modes $A_n$, $\alpha_1$, and $\alpha_2$ of the nonlinear direct lift control.

## Calculation Results

Consider the F-16 combat aircraft[9] with a direct lift control surface. Three basic modes $A_n$, $\alpha_1$, and $\alpha_2$ of the nonlinear direct lift control for the F-16 aircraft are realized by the modes $A$ and $B$ of the decoupling control. For mode $A_n$, $\Delta\alpha = 0$, $\Delta\theta = \Delta\gamma$; for mode $\alpha_1$, $\Delta\gamma = 0$, $\Delta\theta = \Delta\alpha$; for mode $\alpha_2$, $\Delta\theta = 0$, $\Delta y = -\Delta\alpha$. Two modes of extraordinary maneuvers realized by mode $C$ are level yaw, in which the demanded angle of sideslip will be achieved and the initial airspeed and 1 g wings level flight are maintained, and roll angle capture, in which the demanded roll angle will be obtained and at the same time the angle of sideslip and the angle of attack are maintained.

The Mach number and altitude in the following calculations are 0.8 and 6096 m.

The time histories of mode $\alpha_1$ realized by nonlinear decoupling mode $A$ are given by curves 1 in Fig. 1. The constants in

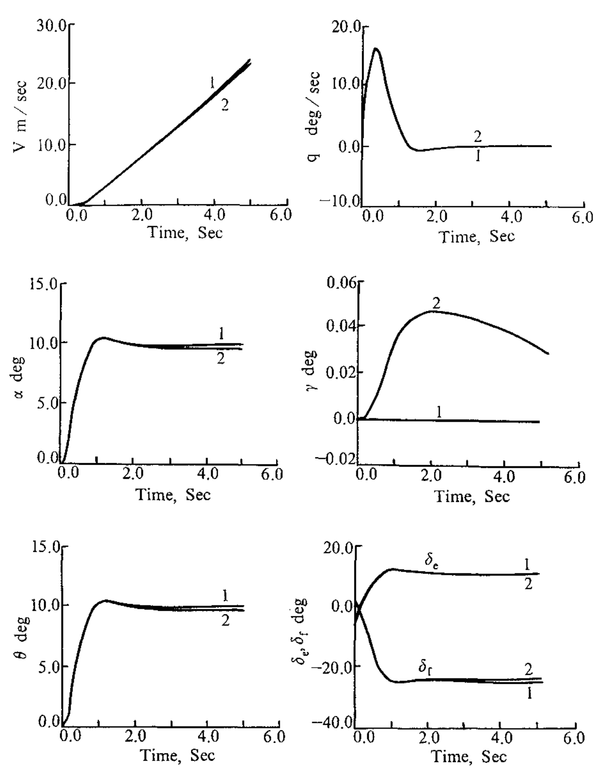


**Fig. 1 Time histories of mode $\alpha_1$ by nonlinear decoupling control: 1) exact solution and 2) approximate solution.**

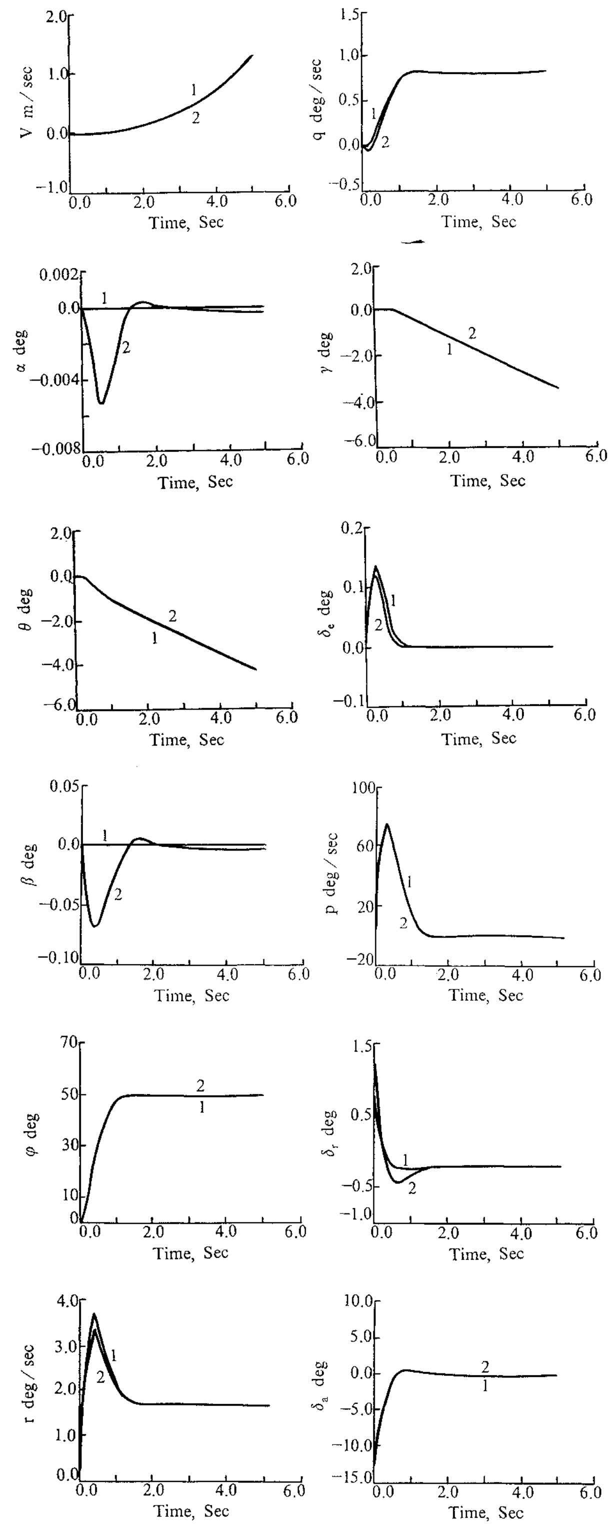


**Fig. 2 Time histories for roll angle capture maneuver by nonlinear decoupling control: 1) exact solution and 2) approximate solution.**

the decoupling mode $A$, which is used to realize mode $\alpha_1$, are given as follows:

$$\alpha_{10} = 2.0, \qquad \alpha_{20} = 12.5, \qquad \alpha_{21} = 5.0$$

$$\lambda_1 = 2.0, \qquad \lambda_2 = 12.5$$

The equilibrium point for the decoupling mode $A$ is given by

$$V = 253 \text{ m/s}, \qquad \alpha = 2.05 \text{ deg}, \qquad q = 0 \text{ deg/s}$$

$$\theta = 2.05 \text{ deg}, \qquad \delta_f = 0.0 \text{ deg}, \qquad \delta_e = 1.11 \text{ deg}$$

$$v_1 = 0.0 \text{ deg}, \qquad v_2 = 2.05 \text{ deg}$$

The step input for $v$ in Fig. 1 is given by $\Delta v = (0 \text{ deg}, 10 \text{ deg})^T$. All curves in Fig. 1 are increments of the state variables and deflection angles relative to the equilibrium point. During the realization of mode $\alpha_1$ the path angle $\gamma$ remains zero throughout. The time histories of mode $\alpha_1$ created by the approximate decoupling control law $\tilde{F}$ and $\tilde{G}$ are presented by curves 2 in Fig. 1. It can be seen from the figure that the error made by the approximation is small and acceptable.

The time histories of the roll angle capture maneuver for the F-16 are illustrated in Fig. 2. The curves 1 in Fig. 2 are the time histories created by the nonlinear decoupling mode $C$. The constants in the decoupling mode $C$, which is used to realize the maneuver, are as follows:

$$\alpha_{10} = 12.5, \qquad \alpha_{11} = 5.0, \qquad \alpha_{20} = 8.0$$

$$\alpha_{21} = 4.0, \qquad \alpha_{30} = 13.0, \qquad \alpha_{31} = 6.0$$

$$\lambda_1 = 12.5, \qquad \lambda_2 = 8.0, \qquad \lambda_3 = 13.0$$

The equilibrium point for the decoupling mode $C$ is given by

$$V = 253 \text{ m/s}, \qquad \alpha = 2.2 \text{ deg}, \qquad q = 0 \text{ deg/s}$$

$$\theta = 2.2 \text{ deg}, \qquad \beta = 0.0 \text{ deg}, \qquad \varphi = 0.0 \text{ deg}$$

$$p = 0 \text{ deg/s}, \qquad r = 0 \text{ deg/s}, \qquad \delta_e = 1.21 \text{ deg}$$

$$\delta_r = 0 \text{ deg}, \qquad \delta_a = 0 \text{ deg}, \qquad v_1 = 2.2 \text{ deg}$$

$$v_2 = 0 \text{ deg}, \qquad v_3 = 0 \text{ deg}$$

The step input for $v$ in Fig. 2 is given by $\Delta v = (0 \text{ deg}, 0 \text{ deg}, 50 \text{ deg})^T$. All curves in Fig. 2 are increments of the state variables and deflection angles relative to the equilibrium point. From the curves it can be seen that the roll angle capture maneuver can be exactly realized by the nonlinear decoupling control law. Cooperative movements of the deflections $\delta_e$, $\delta_a$, and $\delta_r$ are required. Therefore, the decoupling control system is needed to reduce the burden on the pilot during the maneuver.

Curves 2 in Fig. 2 are the time histories produced by the approximate decoupling control law $\tilde{F}$ and $\tilde{G}$ given by Eq. (10). It can be seen from the figures that the error made by the approximation is small and acceptable. The reason for this is that at the calculation state of the example the aerodynamic coefficients are linear. Since $\tilde{F}$ and $\tilde{G}$ in the approximate control law are constant matrices, the amount of numerical calculations can be dramatically reduced.

## Conclusions

Decoupling theory for nonlinear multivariable systems based on differential geometry control theory is applied to design a nonlinear aircraft control system. The influence of the position exchange in components of input and decoupling variable on the decoupling law and closed-loop system is given by Theorems 1 and 2. If the control law for output $y_A = (c_1(x), c_2(x), \ldots, c_n(x))^T$ is known, for example, it is very easy to obtain the control law for output $y_B = (c_2(x), c_1(x), \ldots, c_n(x))^T$ from the control law for output $y_A$ according to Theorem 1.

The calculation of equilibrium points for the nonlinear decoupling system is given in the paper. It can be used to verify the correctness of the nonlinear decoupling control law obtained by numerical computation. For engineering applications, a linear approximation of the aircraft nonlinear decoupling control law is presented.

Modes $A$, $B$, and $C$ of aircraft decoupling control are presented. Modes $A$ and $B$ can be used to realize three basic modes of direct lift control. Mode $C$ can realize roll angle capture and level yaw maneuvers. The simulation results show that the decoupling control modes can realize the modes of nonlinear direct force control and some aircraft agility maneuvers. The error made by the linear approximation is acceptable.

## Acknowledgments

The author is grateful to Kenneth M. Sobel for his invaluable help and advice. Thanks are also due to the anonymous referees whose comments have improved the presentation of the paper considerably.

## References

[1]Snell, S. A., Enns, D. F., and Garrard, W. L., "Nonlinear Inversion Flight Control for a Supermaneuverable Aircraft," *Journal of Guidance, Control, and Dynamics*, Vol. 15, No. 4, 1992, pp. 976–984.

[2]Lane, S. H., and Stengel, R. F., "Flight Control Design Using Non-Linear Inverse Dynamics," *Automatica*, Vol. 24, No. 4, 1988, pp. 471–483.

[3]Singh, S. N., and Schy, A. A.,"Output Feedback Non-Linear Decoupled Control Synthesis and Observer Design for Manoeuvring Aircraft," *International Journal of Control*, Vol. 31, No. 4, 1980, pp. 781–806.

[4]Meyer, G., Su, R., and Hunt, L. R., "Application of Nonlinear Transformations to Automatic Flight Control," *Automatica*, Vol. 20, No. 1, 1984, pp. 103–107.

[5]Singh, S. N., "Control of Nearly Singular Decoupling Systems and Nonlinear Aircraft Maneuver," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 24, No. 6, 1988, pp. 775–784.

[6]Romano, J. J., and Singh, S. N., "I–O Map Inversion, Zero Dynamics and Flight Control," *IEEE Transactions on Aerospace and Electronic Systems*, Vol. 26, No. 6, 1990, pp. 1022–1029.

[7]Azam, M., and Singh, S. N., "Invertibility and Trajectory Control for Nonlinear Maneuvers of Aircraft," *Journal of Guidance, Control, and Dynamics*, Vol. 17, No. 1, 1994, pp. 192–200.

[8]Isidori, A., *Nonlinear Control Systems: An Introduction*, 2nd ed., Springer–Verlag, 1989.

[9]Hoh, R. H., "Development of Handling Quality Criteria for Aircraft with Independent Control of Six Degrees of Freedom," AD/A100045, 1981.